Fractal Analysis of Damage in Rock Under Loading: A Review of Some Recent Studies

M.V.M.S. Rao

Abstract

The fact that rocks undergo deformation and failure under loading is well known. The most important characteristic features of rock deformation prior to final failure are dilatancy, strain hardening and strain softening as evidenced from a large body of stress-strain data. These are attributed to the formation and growth of damage in terms of stress-induced micro-cracks / micro-fractures as inferred from acoustic emission (AE) data that are obtained concurrently with the stress-strain measurements in the laboratory. Furthermore, the size and spatial distribution of damage is found to have a fractal character as observed from the AE signatures (b-values and hypocenter distributions). Interestingly the spatial and temporal distribution of the 'area rock-bursts' that were experienced by the deep underground mines of KGF also were found to have a multifractal character. The fractal structure of these events at widely different scales could be documented and studied well using the power law exponent method for the assessment of size distribution of damage (micro-cracks /micro-fractures) in rock and the correlation dimension method for studying the spatial and temporal distribution of damage in intact as well as jointed rocks, and area rock-bursts of the Champion reef mines of KGF. The results show that the application of fractals to the AE (acoustic emission) and seismic data obtained from rocks under stress can lead to a better identification, description and characterization of the critical state of damage in rock under loading. The above mentioned methods of fractal analysis are briefly outlined and the results of some case studies are presented and reviewed.

Key words: *dilatancy, acoustic emission, b-values, fractal dimension, correlation dimension*

Introduction

Inelastic deformation and failure occurs in rock in a progressive manner due to the formation, growth and coalescence of micro-cracks under compressive loading (Atkinson, 1987). Direct observation of these processes using either optical or electron microscope is quite difficult. However they can be monitored and evaluated, successfully, using the data of Acoustic Emissions (AE), which are transient elastic waves generated due to rapid release of strain energy from materials undergoing deformation and failure under stress (Lockner, 1993). When materials are stressed, AE occur and when the stress is relaxed the emissions cease and no new emissions will occur until the previous maximum stress level has been exceeded. This phenomenon of irreversibility is called Kaiser Effect and it has been helpful in estimating the previously attained stress maximum in rocks under natural geological conditions (Lord and Koerner 1985). Since the deformation and failure of rock are dependent on various factors such as stress, time, temperature and humidity at the tips of the existing and newly created cracks, the use of AE techniques in the frequency range of 50 KHz to 2 MHz has proved to be more advantageous for conducting laboratory tests on rock fracture under environmental conditions (Lockner, 1993).

One of the simplest and most straight forward methods for monitoring the fracture behavior of rock under stress is to count and record the number of AE events and analyse their statistical behavior. Among the signal parameters, the amplitude distribution

National Geophysical Research Institute, Hyderabad – 500 606
(Council of Scientific and Industrial Research)

analysis of AE population and various subsets of it has provided better insights to analyse the various stages of rock fracture in terms of the frequency-magnitude relationship (b-value) of AE following the methods adopted in seismology (Cox and Meredith 1993; Sammonds et al. 1992; Rao, 1996; Rao, 2003; Rao and Prasnna Lakshmi, 2005; Lei et al., 2000; Lei et al., 2005). Furthermore, the technology to locate the sources of AE has vastly Improved and made the spatio-temporal distribution analysis of damage in rock also possible in terms of the AE hypocenter data (Cox and Meredith 1993; Lei et al., 2000; Lei et al., 2005; Rao and Kusonose, 1995; Satoh et al., 1996; Zang et al., 1998; Lei et al., 1998). But in recent years, the application of fractals has gained special significance in order to give a better description and also quantify the size distribution as well as the clustering properties of micro-cracks in rocks undergoing fracture (Hirata et al., 1987; Lei et al., 1992; Shivakumar and Rao, 1997; Shivakumar and Rao, 2000; Rao et al. 1999; Rao et al. 2000). We have deployed the "power law exponent method" to determine the fractal dimension, D of the size distribution, and the "correlation dimension method" to determine the fractal dimension, D_C and fractal analysis of the spatio-temporal distribution of micro-crack damage in rocks subjected to fracture under controlled laboratory conditions. The fractal concepts and measurement methods are outlined and the results obtained are reviewed in this paper.

Laboratory experiments

The rocks tested include some hornblende schists and amphibolites of NX size from the Nundydroog mine (KGF), and basalts, granites and migmatite gneisses of BX size from the basement of Deccan Volcanic Province (DVP), which has been experiencing prolific seismicity. The rock samples of KGF were tested in Japan under incremental and creep loading conditions at constant confining pressure of 30 MPa using a multi-channel AE monitoring and source location system (Lei et al., 2000; Satoh et al., 1996). The tests on basement rock samples of DVP were carried out under uniaxial and triaxial compression using a 2-ch AE monitoring unit and software for processing and analysing the AE statistics and signal parameters (Rao et al. 2008; Rao et al., 2009a; Rao et al., 2009b; Nagraja Rao et al., 2000). The methods to adopt and utilize the AE data for fractal analysis of the stress-induced damage in rock are described in detail in our earlier papers (Shivakumar and Rao, 1997; Shivakumar and Rao, 2000; Rao et al. 1999).

Fractal concepts and measurement methods

Fractal is a new concept that has caught the attention of many researchers in recent years after it was introduced by Mandelbrot (1982) in a geological context. It is used to describe and quantify many irregular and non-smooth objects, phenomena and processes that are found in nature. Mathematically, a fractal is defined as an object whose fractal dimension (which will be defined later) is greater than its topological dimension obtained by the usual Eucledian concepts of length, area etc. Fractal is a scale-invariant structure possessing the property of 'self-similarity'. The self-similarity means that any small portion of a fractal, when magnified by an arbitrary factor, looks the same as the original fractal which in turn is similar to the whole object. In other words it is 'scale-invariant'. All fractals are restricted to a specific range of scales for which scale-invariance applies, and it is important to specify the upper and lower limits as well as the fractal dimension (D) which apply within that range (Mandelbrot, 1982; Grassberger and Procaccia 1983; Turcotte,1986). In the present context, the seismogenic or active fault populations in the earth's upper crust as well as the micro-crack / crack populations producing AE at the laboratory scale are fractals on a wide range. However, the fractal dimension obtained by different methods should not be compared too literally with each other because they reflect different aspects of the scale invariance. For example, the 'power-law exponent (D)' measures the relative proportion of large and small seismogenic faults or micro-cracks/ cracks producing AE (Main et al., 1993) the 'correlation dimension (D_C)' is a measure of the spacing or clustering properties of a set of points representing either earthquake or rock burst epicenter distributions (Hirata 1989; Xie and Pariseau, 1993; Shivakumar et al., 1996), or hypocenter distributions or AE (Hirata et al., 1987; Shivakumar and Rao, 1997, 2000; Lei et al., 2000; Lei et al., 2005), and the 'capacity dimension (D_0)' measures the space filling properties of a fractal set with respect to changes in grid scale (Feder, 1987).

Fractal dimension of size distribution (Power-law exponent)

The number of AE events generated during the laboratory compression tests on rocks is approximately proportional to the number of newly formed micro-cracks, and AE amplitudes that are usually measured in dB are proportional to the size of the crack or crack growth increments (Main et al., 1993). Therefore, the best way to examine the magnitude (amplitude / 20) distribution of AE is to plot the number (N) versus magnitude (M) plots of AE in a logarithmic scale and estimate the b-value (slope of the negative gradient of the power-law) using either the well known Gutenberg-Richter relationship or Aki's equation that are used for the analysis of earthquakes as well as the AE occurring at the laboratory scale for all materials (Main et al., 1993, Rao, 2003). The equations are as follows:

GBR : $\text{Log } N = a\text{-}bM$ .... (1)

AKI : $b = \text{Log}_{10}e \,/\, (M - M_0)$ .... (2)

Where 'a' is a constant, M_0 is the threshold magnitude (or amplitude/ 20) and 'b' is the slope of the straight line portion of the log linear frequency-magnitude distribution plot. The precise values of a and b (scaling constants) are dependent on the rock type and the loading conditions. Generally b is in the interval $0.5 < b < 2.5$. A high AE b-value arises due to a number of relatively small AE events representing new crack formation and slow crack growth, whereas a low b-value indicates faster or unstable crack growth accompanied by relatively high amplitude AE in large numbers. The fractal dimension of the size distribution, D is related to b-value (Main et al, 1993) as follows:

$D = 3b / c$ .... (3).

In general, c = 3/2. Thus the above equation can be rewritten as

$D = 2b$ .... (4).

Fractal dimension of spatial distribution (Number - radius relationship:

The AE or microseismic or seismic event locations construct a spatial distribution of a point set in which a point corresponds to a cracking surface or volume element in physical space. Thus the fractal dimension of the damage evolution process at any given scale can be directly measured from the distribution of the point set (Xie and Pariseau, 1993 29). Considering a sphere with radius r, the total number of events inside this sphere over the distribution can be counted and denoted by M(r). A set of data $M(r_i)$ associated with different radii r_i (i = 1,2,3, ...) can be obtained from fractal geometry. There is a relation between $M(r_i)$ and r_i in the form $M(r) = r^1$ for the line distribution of point set, $M(r) = r^2$ for the plane distribution, and $M(r) = r^3$ for the 3-D (or volume) distribution, and

$M(r) = r^{Dc}$ .... (5)

for a fractal distribution. The above equation is also called the number - radius relation and the fractal dimension, D_c is called the clustering dimension which is equal to the slope of the log M(r) - log(r) plot. In this fractal measurement, the center point of the spheres with different radii r_i is chosen as the mass center of the distribution.

Fractal dimension of spatial distribution (Correlation exponent)

Spatial self-similarity can be demonstrated by examining the distribution of distances between pairs of points in a data set over a range of distances. This has been done on the earthquake scale (Kagan and Knopoff 1980; Hirata, 1989; Hirata et al., 1987 32, 28, 14) and on the laboratory acoustic emission scale (Hirata et al., 1987) using a spatial two-point correlation function. It is given as follows.

$$C(r) = [2N_r (R < r) / n(n-1)]. \quad .... \quad .... \quad .... \quad (6)$$

Where $N_r (R < r)$ is the number of event hypocenter or epicenter pairs with a distance smaller than r, and n is the total number of events. If the distribution of hypocenters or epicenters has self-similar structure, C(r) can be expressed in the form

$$C(r) = r^D \quad .... \quad .... \quad .... \quad .... \quad (7)$$

Where D is a kind of fractal dimension called the correlation exponent that gives the lower limit of the Hausdorff dimensions. This method was adopted by us for investigating the fractal character of the AE hypocenter distributions of rocks at the laboratory scale (Shivakumar and Rao 1997; Lei et al., 2005; Rao and Prasanna, 2005).

Multi-fractal dimension

A multi-fractal is heterogeneous and is interwoven with infinitely many subfractal sets of different dimensions. The generalized dimension, D_q for them can be estimated using the space filling properties of fractal objects and the power-law exponent method as described in detail by Shivakumar & Rao (2000). The equation for computing the multi-fractal dimension is as follows:

$$Cq (r) = r^{Dq} \quad .... \quad .... \quad .... \quad .... \quad (8)$$

where Cq (r) is the generalized correlation integral function, r is a local density function, and q is a parameter for making the iteration to estimate the probability function of the spatial distribution of points.

Case studies

Multi-fractal structure of area rock bursts of Champion Reef mine

The first successful attempts in India on fractal analysis of damage in rock under in situ conditions were carried out by Shivakumar et al. (1996). The in-put data for it were the hypocenter distributions of three major Area Rock Bursts (ARB) and a series of rock bursts that followed in quick succession in the form of clusters in the region of mining activity and/ or in the abandoned areas of mining (old workings) of Champion Reef mine, KGF. The spatial distribution of all the three area rock bursts has revealed that they are heterogeneous fractals with generalized dimension values of 2.10, 1.58 and 1.95 respectively. Among them, the ARB-I & ARB-II occurred at depths below 400 ft. Whereas the ARB-II occurred in a highly stressed region at a depth of ~ 10,000 ft where prominent geological features such as Mysore north fault, pegmatite intrusions, a vertical dyke and folded nature of the lode along with the working stopes were found to be responsible for a smaller fractal dimension (1.58) and with a less degree of heterogeneity (0.37) compared to the other two (Shivakumar et al., 1996; Shivakumar and Rao 2000; Karekal et al., 2005).

Fractal character of micro-seismic activity

The spatial distribution and event count rates of microsesimic events associated with two rock bursts that occurred at the 98 and 103 levels of Champion reef mine were analysed by applying the principles of fractal geometry (Rao et al., 2001). The Correlation dimension was found to decrease from 2.5 to 0.2 in one case and from 2.75 to 1.2 in the other case with striking variations among the source locations prior to and following the rock bursts (Rao et al., 2000).

Fractal analysis of spatial distribution of microcrack damage in Kolar amphibolites

The hypocenter data of 1800 AE events recorded during the triaxial compression and creep tests at 30 MPa confining pressure in the GSJ laboratory on a jointed amphiboiite rock sample of the Nundydroog mine (Satoh et al., 1996) were processed and analysed (Shivakumar and Rao, 1997; Shivakumar and Rao, 2000). The D_c values computed from the slopes of log C (r) versus log (r) during the primary, secondary and tertiary stages of creep were found to be 0.67, 1.07 and 1.82 respectively. These observations indicate that the microcracks which concentrated more on the joint plane during the incremental loading and primary creep weakened the material resulting in low D_c of 0.67. Subsequently, the microcracking activity during secondary and tertiary creep regimes shifted on to the eventual fracture plane with diffused AE activity. These observations are quite useful for the interpretation of seismic activity associated with fault zones in rock masses.

Fractal analysis of size distribution of microcrack populations in rock

The AE b-value (2D) data obtained during some of the controlled laboratory tests performed under traixial compression and creep (Lei et al. 2000) has been very useful to investigate the fault nucleation and its quasi-static growth in intact brittle rocks such as hornblende schists of Nundydroog mine (Lei et al 2000), and granitic rocks of different grain size (Lei et al 2005). Further the mechanics of brittle deformation and crack growth could be inferred from AE statistics because the number of AE events is proportional to the number of growing cracks and because AE amplitudes are proportional to the length of crack growth increments in rock (Lei et al 2000; Lei et al 2005). The AE b-value (or D) data obtained during the uniaxial compression tests of granites and the basement rocks of DVP show that b is ~ 1.0 during a large portion of the loading regime. As the impending failure approaches in the rock, the AE b-value not only decreases sharply to as low as 0.5 (or D = 1.0) for hard rocks but also shows short-term anomalies in terms of the underlying physical processes of crack growth in rocks containing weak planes and grain size anomalies (Rao et al., 2009a & 2009b).

Conclusions

1. The fractal character of rock fracture and seismic energy release at different scales has been investigated using the AE, microsesimic and seismic data of the rocks and underground mines of the Kolar Gold Fields using the power-law exponent method and correlation dimension method.
2. Fractal analysis of AE data accompanying rock fracture at the laboratory scale can yield a better description and quantification of size and spatial distribution of damage evolution in intact as well as jointed rocks under a variety of loading conditions. The state of criticality of rock under stress can be more accurately identified and tracked in terms of the fractal dimension for a better prognosis of rock failure and to predict and control catastrophic rock failures.
3. Multi-fractal analysis of the spatial distribution of three major area rock bursts of the Champion Reef mine indicated that they are bounded by different heterogeneous stress fields associated with different mining and geological conditions.

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